Advanced Higher
Course
Specification

Advanced Higher Mathematics

Course code:	C847 77
Course assessment code:	X847 77
SCQF:	level 7 (32 SCQF credit points)
Valid from:	session 2019–20

This document provides detailed information about the course and course assessment to ensure consistent and transparent assessment year on year. It describes the structure of the course and the course assessment in terms of the skills, knowledge and understanding that are assessed.

This document is for teachers and lecturers and contains all the mandatory information required to deliver the course.

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Contents

Course overview

This course consists of 32 SCQF credit points, which includes time for preparation for course assessment. The notional length of time for candidates to complete the course is 160 hours.

The course assessment has two components.

Component	Marks	Duration
Component 1: question paper 1 (non-calculator)	35	1 hour
Component 2: question paper 2	80	2 hours and 30 minutes

Recommended entry	Progression
Entry to this course is at the discretion of the centre. Candidates should have achieved the Higher Mathematics course or equivalent qualifications and/or experience prior to starting this course.	♦ other qualifications in mathematics or related areas ♦ further study, employment and/or training

Conditions of award

The grade awarded is based on the total marks achieved across both course assessment components.

Course rationale

National Courses reflect Curriculum for Excellence values, purposes and principles. They offer flexibility, provide time for learning, focus on skills and applying learning, and provide scope for personalisation and choice.

Every course provides opportunities for candidates to develop breadth, challenge and application. The focus and balance of assessment is tailored to each subject area.

Learning mathematics develops logical reasoning, analysis, problem-solving skills, creativity, and the ability to think in abstract ways. It uses a universal language of numbers and symbols, which allows us to communicate ideas in a concise, unambiguous, and rigorous way.

The course develops existing knowledge and introduces advanced mathematical techniques, which are critical to successful progression beyond Advanced Higher level in Mathematics and many other curriculum areas. The skills, knowledge and understanding in the course also support learning in technology, health and wellbeing, science, and social studies.

Purpose and aims

Mathematics is important in everyday life. It helps us to make sense of the world we live in and to manage our lives.

Using mathematics enables us to model real-life situations and make connections and informed predictions. It equips us with the skills we need to interpret and analyse information, simplify and solve problems, assess risk and make informed decisions.

The course aims to:

- motivate and challenge candidates by enabling them to select and apply complex mathematical techniques in a variety of mathematical situations
- extend candidates' skills in problem solving and logical thinking
- clarify candidates' thinking through the process of rigorous proof
- allow candidates to interpret, communicate, and manage information in mathematical form, skills which are vital to scientific and technological research and development
- develop confidence in the subject and a positive attitude towards further study in mathematics and the use of mathematics in employment
- deliver in-depth study of mathematical concepts and the ways in which mathematics describes our world
- deepen candidates' skills in using mathematical language and exploring advanced mathematical ideas

Who is this course for?

This course is particularly suitable for candidates who:

- have demonstrated an aptitude for Higher Mathematics
- are interested in developing mathematical techniques to use in further study or in the workplace

Course content

The Advanced Higher Mathematics course develops, deepens and extends the mathematical skills necessary at this level and beyond.

Throughout this course, candidates acquire and apply operational skills necessary for exploring complex mathematical ideas. They select and apply mathematical techniques and develop their understanding of the interdependencies within mathematics.

Candidates develop mathematical reasoning skills and gain experience in making informed decisions.

Skills, knowledge and understanding

Skills, knowledge and understanding for the course

The following provides a broad overview of the subject skills, knowledge and understanding developed in the course:

- using mathematical reasoning skills to think logically, provide justification, and solve problems
- knowledge and understanding of a range of complex concepts
- selecting and applying complex operational skills
- using reasoning skills to interpret information and complex mathematical models
- effectively communicating solutions in a variety of contexts
- explaining and justifying concepts through the idea of rigorous proof
- thinking creatively

Skills, knowledge and understanding for the course assessment

The following provides details of skills, knowledge and understanding sampled in the course assessment.

Calculus	
Skill	**Explanation**
Differentiating exponential and natural logarithmic functions	♦ differentiating functions involving $e^x, \ln x$
Differentiating functions using the chain rule	♦ applying the chain rule to differentiate the composition of at most three functions
Differentiating functions given in the form of a product and in the form of a quotient	♦ differentiating functions of the form $f(x)g(x)$ and $\frac{f(x)}{g(x)}$ ♦ knowing the definitions and applying the derivatives of $\tan x$, $\cot x$, $\sec x$ and $\operatorname{cosec} x$ ♦ deriving and using the derivatives of $\tan x$, $\cot x$, $\sec x$ and $\operatorname{cosec} x$ ♦ differentiating functions that require more than one application or combination of applications of chain rule, product rule, and quotient rule ♦ applying $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ where appropriate
Differentiating inverse trigonometric functions	♦ differentiating expressions of the form $\sin^{-1}[f(x)]$, $\cos^{-1}[f(x)]$, $\tan^{-1}[f(x)]$
Finding the derivative where relationships are defined implicitly	♦ using differentiation to find the first derivative of a relationship defined implicitly, including in context ♦ using differentiation to find the second derivative of a relationship defined implicitly ♦ using logarithmic differentiation; recognising when it is appropriate in extended products, quotients, and in functions where the variable occurs in an index ♦ applying differentiation to related rates in problems where the relationship may or may not be given

Calculus	
Skill	**Explanation**
Finding the derivative where relationships are defined parametrically	♦ using differentiation to find the first derivative of a relationship defined parametrically ♦ applying parametric differentiation to motion in a plane, including instantaneous speed ♦ using differentiation to find the second derivative of a relationship defined parametrically
Applying differentiation to problems in context	♦ applying differentiation to problems in context ♦ applying differentiation to optimisation
Integrating expressions using standard results	♦ using $\int e^{ax+b}\,dx$, $\int \frac{dx}{ax+b}$, $\int \sec^2(ax+b)\,dx$, $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$, $\int \frac{1}{a^2+x^2}\,dx$ ♦ recognising and integrating expressions of the form $\int g(f(x))f'(x)\,dx$ and $\int \frac{f'(x)}{f(x)}\,dx$ ♦ using partial fractions to integrate proper or improper rational functions
Integrating by substitution	♦ integrating where the substitution is given
Integrating by parts	♦ using integration by parts with one or more applications
Applying integration to problems in context	♦ applying integration to volumes of revolution, where the volume generated is by the rotation of the area under a single curve about the x-axis or y-axis ♦ applying integration to the evaluation of areas, including integration with respect to y ♦ applying integration to problems in context
Solving first-order differential equations with variables separable	♦ finding general and particular solutions to equations that can be written in the form $\frac{dy}{dx} = g(x)h(y)$ or $\frac{dy}{dx} = \frac{g(x)}{h(y)}$

Calculus	
Skill	**Explanation**
Solving first-order linear differential equations using an integrating factor	♦ finding general and particular solutions to equations that can be written in the form $\frac{dy}{dx}+P(x)y=f(x)$
Solving second-order differential equations	♦ finding general and particular solutions of second-order linear ordinary differential equations of the form $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ (homogeneous) $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$ (non-homogeneous) where the roots of the auxiliary equation may be: ▪ real and distinct ▪ real and equal ▪ complex conjugates

Algebra, proof and number theory	
Skill	**Explanation**
Decomposing a rational function into a sum of partial fractions (denominator of degree at most three)	♦ decomposing a proper rational function as a sum of partial fractions where the denominator may contain distinct linear factors, an irreducible quadratic factor, or a repeated linear factor ♦ reducing an improper rational function to a polynomial and a proper rational function by division or otherwise
Finding the asymptotes to the graphs of rational functions	♦ finding the vertical asymptote(s) to the graph of a rational function ♦ finding the non-vertical asymptote to the graph of a rational function
Investigating features of graphs and sketching graphs of functions	♦ investigating points of inflection ♦ investigating features of graphs: ▪ points of inflection ▪ stationary points ▪ domain and range ▪ odd, even, or neither ▪ continuous or discontinuous ▪ extrema of functions: the maximum and minimum values of a continuous function f defined on a closed interval $[a,b]$ can occur at stationary points, end points, or points where f' is not defined ♦ sketching graphs using features given or obtained ♦ sketching related functions: ▪ modulus functions ▪ inverse functions ▪ functions differentiated ▪ translations and reflections
Expanding expressions using the binomial theorem	♦ using the binomial theorem $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, for $r, n \in \mathbb{N}$ to expand an expression of the form $(ax^p + by^q)^n$, where $a,b \in \mathbb{Q}$; $p,q \in \mathbb{Z}$; $n \le 7$ ♦ using the general term for a binomial expansion, finding a specific term in an expression

Algebra, proof and number theory	
Skill	**Explanation**
Finding the general term and summing arithmetic and geometric progressions	♦ applying the rules of sequences and series to find: ▪ the *n*th term ▪ the sum to *n* terms ▪ common difference of arithmetic sequences ▪ common ratio of geometric sequences ♦ determining the sum to infinity of geometric series ♦ determining the condition for a geometric series to converge
Applying summation formulae	♦ knowing and using sums of certain series, and other straightforward results and combinations of these
Using the Maclaurin expansion to find specified terms of the power series for simple functions	♦ using the Maclaurin expansion to find a power series for simple functions ♦ combining Maclaurin expansions to find a power series
Disproving a conjecture by providing a counterexample	♦ disproving a conjecture by providing a counterexample ♦ knowing and using the symbols $\exists$ (there exists) and $\forall$ (for all) ♦ giving the negation of a statement
Using indirect or direct proof in straightforward examples	♦ proving a statement by contradiction ♦ using proof by contrapositive ♦ using direct proof in straightforward examples
Using proof by induction	♦ using proof by induction
Using Euclid's algorithm to find the greatest common divisor of two positive integers	♦ using Euclid's algorithm to find the greatest common divisor of two positive integers, for example using the division algorithm repeatedly ♦ expressing the greatest common divisor (of two positive integers) as a linear combination of the two ♦ expressing integers in bases other than 10 ♦ knowing and using the fundamental theorem of arithmetic

Matrices, vectors and complex numbers	
Skill	**Explanation**
Using Gaussian elimination to solve a 3 × 3 system of linear equations	♦ finding the solution to a system of equations $A\boldsymbol{x} = \mathbf{b}$, where A is a 3 × 3 matrix and where the solution is unique — candidates should understand the term 'augmented matrix' ♦ showing that a system of equations has no solutions (inconsistency) ♦ showing that a system of equations has an infinite number of solutions (redundancy) ♦ comparing the solutions of related systems of two equations in two unknowns and recognising ill-conditioning
Understanding and using matrix algebra	♦ performing matrix operations (at most order three): addition, subtraction, multiplication by a scalar, multiplication of matrices ♦ knowing and applying the properties of matrix addition and multiplication: ▪ $A+B=B+A$ (addition is commutative) ▪ $AB \neq BA$ (multiplication is not commutative in general) ▪ $(A+B)+C=A+(B+C)$ (associativity) ▪ $(AB)C=A(BC)$ (associativity) ▪ $A(B+C)=AB+AC$ (addition is distributive over multiplication) ♦ knowing and applying key properties of the transpose, the identity matrix, and inverse: ▪ $(a_{ij})'_{m\times n}=(a_{ji})_{n\times m}$ (rows and columns interchange) ▪ $(A')'=A$ ▪ $(A+B)'=A'+B'$ ▪ $(AB)'=B'A'$ ▪ A square matrix A is orthogonal if $A'A=AA'=I$ ▪ The $n \times n$ identity matrix I_n for any square matrix A , $AI_n = I_n A = A$ ▪ $B=A^{-1}$ if $AB=BA=I$ ▪ $(AB)^{-1}=B^{-1}A^{-1}$

Matrices, vectors and complex numbers	
Skill	**Explanation**
Calculating the determinant of a matrix	♦ finding the determinant of a 2 × 2 matrix and a 3 × 3 matrix ♦ determining whether a matrix is singular ♦ knowing and applying $\det(AB) = \det A \det B$
Finding the inverse of a matrix	♦ knowing and using the inverse of a 2 × 2 matrix ♦ finding the inverse of a 3 × 3 matrix
Using transformation matrices	♦ using 2 × 2 matrices to carry out geometric transformations in the plane — the transformations should include rotations, reflections, and dilatations ♦ applying combinations of transformations
Calculating a vector product	♦ using a vector product method in three dimensions ♦ evaluating the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
Working with lines in three dimensions	♦ finding the equation of a line in parametric, symmetric, or vector form, given suitable defining information ♦ finding the angle between two lines in three dimensions ♦ determining whether or not two lines intersect and, where possible, finding the point of intersection
Working with planes	♦ finding the equation of a plane in vector, parametric, or Cartesian form ♦ finding the point of intersection of a plane with a line that is not parallel to the plane ♦ determining the intersection of two or three planes ♦ finding the angle between a line and a plane, or between two planes
Performing algebraic operations on complex numbers	♦ performing the operations of addition, subtraction, multiplication, and division ♦ finding the square root ♦ finding the roots of a cubic or quartic equation with real coefficients when one complex root is given ♦ solving equations involving complex numbers

Matrices, vectors and complex numbers	
Skill	**Explanation**
Performing geometric operations on complex numbers	♦ plotting complex numbers in the complex plane (an Argand diagram) ♦ knowing the definition of modulus and argument of a complex number ♦ converting a given complex number from Cartesian to polar form and vice-versa ♦ using de Moivre's theorem with integer and fractional indices ♦ applying de Moivre's theorem to multiple angle trigonometric formulae ♦ applying de Moivre's theorem to find the *n*th roots of a complex number ♦ interpreting geometrically certain equations or inequalities in the complex plane by sketching or describing a straight line or circle that represents the locus of points that satisfy a given equation or inequality

Skills, knowledge and understanding included in the course are appropriate to the SCQF level of the course. The SCQF level descriptors give further information on characteristics and expected performance at each SCQF level, and are available on the SCQF website.

Skills for learning, skills for life and skills for work

This course helps candidates to develop broad, generic skills. These skills are based on SQA's Skills Framework: Skills for Learning, Skills for Life and Skills for Work and draw from the following main skills areas:

2 Numeracy
2.1 Number processes
2.2 Money, time and measurement
2.3 Information handling

5 Thinking skills
5.3 Applying
5.4 Analysing and evaluating

You must build these skills into the course at an appropriate level, where there are suitable opportunities.

Course assessment

Course assessment is based on the information in this course specification.

The course assessment meets the purposes and aims of the course by addressing:

- breadth — drawing on knowledge and skills from across the course
- challenge — requiring greater depth or extension of knowledge and/or skills
- application — requiring application of knowledge and/or skills in practical or theoretical contexts as appropriate

This enables candidates to:

- use mathematical reasoning skills to think logically, provide justification, and solve problems
- use a range of complex concepts
- select and apply complex operational skills
- use reasoning skills to interpret information and use complex mathematical models
- effectively communicate solutions in a variety of mathematical contexts
- explain and justify concepts through the idea of rigorous proof
- think creatively

Course assessment structure: question paper

Question paper 1 (non-calculator) 35 marks

This question paper allows candidates to demonstrate the application of mathematical skills, knowledge and understanding from across the course. Candidates must not use a calculator.

This question paper gives candidates an opportunity to apply numerical, algebraic, geometric, trigonometric, calculus, and reasoning skills specified in the 'Skills, knowledge and understanding for the course assessment' section.

This question paper has 35 marks out of a total of 115 marks for the course assessment. It consists of short-answer and extended-response questions.

Setting, conducting and marking the question paper

This question paper is set and marked by SQA, and conducted in centres under conditions specified for external examinations by SQA.

Candidates have 1 hour to complete this question paper.

Question paper 2 80 marks

This question paper assesses mathematical skills. Candidates may use a calculator.

This question paper gives candidates an opportunity to apply numerical, algebraic, geometric, trigonometric, calculus, and reasoning skills specified in the ‘Skills, knowledge and understanding for the course assessment’ section.

Using a calculator can facilitate these skills and allow more opportunity for application and reasoning. When solving problems, candidates typically use calculators to perform calculations that are more complex.

This question paper has 80 marks out of a total of 115 marks for the course assessment. It consists of short-answer and extended-response questions.

Setting, conducting and marking the question paper

This question paper is set and marked by SQA, and conducted in centres under conditions specified for external examinations by SQA.

Candidates have 2 hours and 30 minutes to complete this question paper.

Specimen question papers for Advanced Higher courses are published on SQA’s website. These illustrate the standard, structure and requirements of the question papers. The specimen papers also include marking instructions.

Grading

Candidates’ overall grades are determined by their performance across the course assessment. The course assessment is graded A–D on the basis of the total mark for both course assessment components.

Grade description for C

For the award of grade C, candidates will typically have demonstrated successful performance in relation to the skills, knowledge and understanding for the course.

Grade description for A

For the award of grade A, candidates will typically have demonstrated a consistently high level of performance in relation to the skills, knowledge and understanding for the course.

Equality and inclusion

This course is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

Guidance on assessment arrangements for disabled candidates and/or those with additional support needs is available on the assessment arrangements web page: www.sqa.org.uk/assessmentarrangements.

Further information

- Advanced Higher Mathematics subject page
- Assessment arrangements web page
- Building the Curriculum 3–5
- Guide to Assessment
- Guidance on conditions of assessment for coursework
- SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work
- Coursework Authenticity: A Guide for Teachers and Lecturers
- Educational Research Reports
- SQA Guidelines on e-assessment for Schools
- SQA e-assessment web page
- SCQF website: framework, level descriptors and SCQF Handbook

Appendix 1: course support notes

Introduction

These support notes are not mandatory. They provide advice and guidance to teachers and lecturers on approaches to delivering the course. Please read these course support notes in conjunction with the course specification and the specimen question papers.

Approaches to learning and teaching

Approaches to learning and teaching should be engaging, with opportunities for personalisation and choice built in where possible. These could include:

- project-based tasks such as investigating the graphs of related functions, which could include using calculators or other technologies
- a mix of collaborative, co-operative or independent tasks that engage candidates
- solving problems and thinking critically
- explaining thinking and presenting strategies and solutions to others
- using questioning and discussion to encourage candidates to explain their thinking and to check their understanding of fundamental concepts
- making links in themes which cut across the curriculum to encourage transferability of skills, knowledge and understanding — including with technology, geography, sciences, social subjects, and health and wellbeing
- debating and discussing topics and concepts so that candidates can demonstrate skills in constructing and sustaining lines of argument to provide challenge, enjoyment, breadth, and depth in their learning
- drawing conclusions from complex information
- using sophisticated written and/or oral communication and presentation skills to present information
- using technological and media resources, for example web-based resources and video clips
- using real-life contexts and experiences familiar and relevant to candidates to hone and exemplify skills, knowledge and understanding

You should support candidates by having regular discussions with them and giving them regular feedback. For group activities, candidates could also receive feedback from their peers.

You should, where possible, provide opportunities for candidates to personalise their learning and give them choices about learning and teaching approaches. The flexibility in Advanced Higher courses and the independence with which candidates carry out the work lend themselves to this.

You should use inclusive approaches to learning and teaching. There may be opportunities to contextualise approaches to learning and teaching to Scottish contexts in this course. You could do this through mini-projects or case studies.

Preparing for course assessment

The course assessment focuses on breadth, challenge and application. Candidates draw on and extend the skills they have learned during the course. These are assessed through two question papers: one non-calculator and another in which candidates can use a calculator.

To help candidates prepare for the course assessment, they should have the opportunity to:

- analyse a range of real-life problems and situations involving mathematics
- select and adapt appropriate mathematical skills
- apply mathematical skills with and without the aid of a calculator
- determine solutions
- explain solutions and/or relate them to context
- present mathematical information appropriately

The question papers assess a selection of knowledge and skills acquired during the course, and provide opportunities for candidates to apply skills in a wide range of situations, some of which may be new.

Before the course assessment, candidates may benefit from responding to short-answer questions and extended-response questions.

Developing skills for learning, skills for life and skills for work

You should identify opportunities throughout the course for candidates to develop skills for learning, skills for life and skills for work.

Candidates should be aware of the skills they are developing and you can provide advice on opportunities to practise and improve them.

SQA does not formally assess skills for learning, skills for life and skills for work.

There may also be opportunities to develop additional skills depending on the approach centres use to deliver the course. This is for individual teachers and lecturers to manage.

Some examples of potential opportunities to practise or improve these skills are provided in the following table.

SQA skills for learning, skills for life and skills for work framework definition	Suggested approaches for learning and teaching
Numeracy is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. It is also the ability to understand the results.	Candidates could: ♦ develop their numerical skills throughout the course, for example by using surds in differential and integral calculus, and solving equations using Gaussian elimination ♦ use numbers to solve contextualised problems involving other STEM subjects ♦ manage problems, tasks and case studies involving numeracy by analysing the context, carrying out calculations, drawing conclusions, and making deductions and informed decisions
Applying is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.	Candidates could: ♦ apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life contexts ♦ think creatively to adapt strategies to suit the given problem or situation ♦ show and explain their thinking to determine their level of understanding ♦ think about how they are going to tackle problems or situations, decide which skills to use, and then carry out the calculations necessary to complete the task, for example solving problems using related rates of change
Analysing and evaluating is the ability to identify and weigh-up the features of a situation or issue and to use judgement to come to a conclusion. It includes reviewing and considering any potential solutions.	Candidates could: ♦ identify which real-life tasks or situations require the use of mathematics ♦ interpret the results of their calculations and draw conclusions; conclusions drawn could be used to form the basis of making choices or decisions ♦ identify and analyse situations involving mathematics that are of personal interest

During the course, candidates have opportunities to develop their literacy skills and employability skills.

Literacy skills are particularly important, as these skills allow candidates to access, engage in and understand their learning, and to communicate their thoughts, ideas and opinions. The course provides candidates with the opportunity to develop their literacy skills by analysing real-life contexts and communicating their thinking by presenting mathematical information in a variety of ways. This could include the use of numbers, formulae, diagrams, graphs, symbols and words.

Employability skills are the personal qualities, skills, knowledge, understanding and attitudes required in changing economic environments. Candidates can apply the mathematical operational and reasoning skills developed in this course in the workplace. The course provides them with the opportunity to analyse a situation, decide which mathematical strategies to apply, work through those strategies effectively, and make informed decisions based on the results.

Appendix 2: skills, knowledge and understanding with suggested learning and teaching contexts

The first two columns are identical to the tables of ‘Skills, knowledge and understanding for the course assessment’ in the course specification.

The third column gives examples of where the skills could be used in individual activities or pieces of work.

Calculus		
Skill	**Explanation**	**Examples**
Differentiating exponential and natural logarithmic functions	♦ differentiating functions involving $e^x, \ln x$	For example: ♦ $y = e^{3x}$ ♦ $f(x) = \ln(x^3 + 2)$
Differentiating functions using the chain rule	♦ applying the chain rule to differentiate the composition of at most three functions	For example: ♦ $y = \sqrt{e^{x^2} + 4}$ ♦ $f(x) = \sin^3(2x - 1)$ Candidates would benefit from exposure to formal proofs of differentiation.

Calculus		
Skill	**Explanation**	**Examples**
Differentiating functions given in the form of a product and in the form of a quotient	♦ differentiating functions of the form $f(x)g(x)$ and $\frac{f(x)}{g(x)}$	For example: ♦ $y = 3x^4 \sin x$ ♦ $f(x) = x^2 \ln x,\ x > 0$ ♦ $y = \frac{2x-5}{3x^2+2}$ ♦ $f(x) = \frac{\cos x}{e^x}$ Candidates should consider different ways of expressing their answers.
	♦ knowing the definitions and applying the derivatives of $\tan x$, $\cot x$, $\sec x$ and $\operatorname{cosec} x$ ♦ deriving and using the derivatives of $\tan x$, $\cot x$, $\sec x$ and $\operatorname{cosec} x$	
	♦ differentiating functions that require more than one application or combination of applications of chain rule, product rule, and quotient rule	For example: ♦ $y = e^{2x} \tan 3x$ ♦ $y = \ln\lvert -3 + \sin 2x \rvert$

Calculus		
Skill	**Explanation**	**Examples**
	♦ applying $\frac{dy}{dx}=\frac{1}{\left(\frac{dx}{dy}\right)}$ where appropriate	♦ $y=\frac{\sec 2x}{e^{3x}}$ ♦ $y=\frac{\tan 2x}{1+3x^2}$ Learning and teaching contexts could include applying differentiation to rates of change, such as rectilinear motion and optimisation.
Differentiating inverse trigonometric functions	♦ differentiating expressions of the form $\sin^{-1}[f(x)]$, $\cos^{-1}[f(x)]$, $\tan^{-1}[f(x)]$	For example: ♦ linking with the graphs of these functions ♦ making reference to $f^{-1}(f(x))=x\Rightarrow(f^{-1})'(f(x))f'(x)=1\Rightarrow(f^{-1})'(f(x))=\frac{1}{f'(x)}$
Finding the derivative where relationships are defined implicitly	♦ using differentiation to find the first derivative of a relationship defined implicitly, including in context ♦ using differentiation to find the second derivative of a relationship defined implicitly ♦ using logarithmic differentiation; recognising when it is appropriate in extended products, quotients, and in functions where the variable occurs in an index	For example: ♦ $x^3y+xy^3=4$ ♦ $y=\frac{x^2\sqrt{7x-3}}{1+x}$ ♦ $y=2^x$, $y=x^{\tan x}$ ♦ link with obtaining the derivatives of inverse trigonometric functions

Calculus		
Skill	**Explanation**	**Examples**
	♦ applying differentiation to related rates in problems where the relationship may or may not be given	For example: ♦ the 'falling ladder' problem ♦ spherical balloons being inflated (or deflated) at a given rate ♦ the rate at which the depth of coffee in a conical filter changes, for example $V = \frac{1}{3}\pi r^2 h$; given $\frac{dh}{dt}$, find $\frac{dV}{dt}$ ♦ velocity and acceleration: $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$
Finding the derivative where relationships are defined parametrically	♦ using differentiation to find the first derivative of a relationship defined parametrically ♦ applying parametric differentiation to motion in a plane, including instantaneous speed ♦ using differentiation to find the second derivative of a relationship defined parametrically	Candidates should understand the geometrical importance of parametric equations. If the position is given by $x = f(t)$, $y = g(t)$, then: ♦ velocity components are given by $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$ ♦ speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$
Applying differentiation to problems in context	♦ applying differentiation to problems in context ♦ applying differentiation to optimisation	For example, a particle moves a distance s metres in t seconds. The distance travelled by the particle is given by $s = 2t^3 - \frac{23}{2}t^2 + 3t + 5$. Find the acceleration of the particle after 4 seconds.

Calculus		
Skill	**Explanation**	**Examples**
Integrating expressions using standard results	♦ using $\int e^{ax+b}\,dx$, $\int \frac{dx}{ax+b}$, $\int \sec^2(ax+b)\,dx$, $\int \frac{1}{\sqrt{a^2-x^2}}dx$, $\int \frac{1}{a^2+x^2}dx$	For example: ♦ $\int e^{5x-7}\,dx$, $\int \frac{dx}{2x-4}$ Link this with obtaining the derivatives of inverse trigonometric functions.
	♦ recognising and integrating expressions of the form $\int g(f(x))f'(x)dx$ and $\int \frac{f'(x)}{f(x)}dx$	For example: ♦ $\int \cos^3 x \sin x\ dx$ ♦ $\int xe^{x^2}\ dx$ ♦ $\int_0^2 \frac{2x}{x^2+3}dx$ ♦ $\int \frac{\cos x}{(5+2\sin x)}dx$
	♦ using partial fractions to integrate proper or improper rational functions	Candidates should know how to deal with definite or indefinite integrals, as required.

Calculus		
Skill	**Explanation**	**Examples**
Integrating by substitution	♦ integrating where the substitution is given	For example, use the substitution $u = \ln x$ to obtain $\int \frac{1}{x \ln x} dx$, where $x > 1$. Candidates should know how to deal with definite or indefinite integrals, as required.
Integrating by parts	♦ using integration by parts with one or more applications	Derive from the product rule, for example: ♦ $\int x \sin x dx$ (single application) ♦ $\int x^2 e^{3x} dx$ (repeated applications) ♦ $\int e^x \sin x dx$ (cyclic integration) ♦ $\int \ln x dx$ (by considering $\ln x$ as $1.\ln x$) Candidates should know how to deal with definite or indefinite integrals, as required.
Applying integration to problems in context	♦ applying integration to volumes of revolution, where the volume generated is by the rotation of the area under a single curve about the x-axis or y-axis ♦ applying integration to the evaluation of areas, including integration with respect to y ♦ applying integration to problems in context	For example, given velocity, use integration to find displacement.

Calculus		
Skill	**Explanation**	**Examples**
Solving first-order differential equations with variables separable	♦ finding general and particular solutions to equations that can be written in the form $\frac{dy}{dx} = g(x)h(y)$ or $\frac{dy}{dx} = \frac{g(x)}{h(y)}$	Candidates should be aware that differential equations arise in modelling of physical situations (for example electrical circuits, population growth, Newton's law of cooling) and given further information, they can obtain a particular solution.
Solving first-order linear differential equations using an integrating factor	♦ finding general and particular solutions to equations that can be written in the form $\frac{dy}{dx} + P(x)y = f(x)$	Candidates could practise rearranging into standard form. Examples of this include acceleration under gravity with air resistance, and simple electronic circuits.
Solving second-order differential equations	♦ finding general and particular solutions of second-order linear ordinary differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ (homogeneous) $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ (non-homogeneous) where the roots of the auxiliary equation may be: ▪ real and distinct ▪ real and equal ▪ complex conjugates	Context applications could include the motion of a spring, both with and without a damping term.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Decomposing a rational function into a sum of partial fractions (denominator of degree at most three)	♦ decomposing a proper rational function as a sum of partial fractions where the denominator may contain distinct linear factors, an irreducible quadratic factor, or a repeated linear factor ♦ reducing an improper rational function to a polynomial and a proper rational function by division or otherwise	This is required for integration of rational functions and useful in the context of differentiation. For example: ♦ $\frac{7x+1}{x^2+x-6}=\frac{A}{x+3}+\frac{B}{x-2}$ ♦ $\frac{5x^2-x+6}{x^3+3x}=\frac{A}{x}+\frac{Bx+C}{x^2+3}$ ♦ $\frac{3x+10}{(x-1)(x+3)^2}=\frac{A}{x-1}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$ ♦ $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$ ♦ $\frac{x^2+3x}{x^2-4}$ ♦ graph sketching when asymptotes are present
Finding the asymptotes to the graphs of rational functions	♦ finding the vertical asymptote(s) to the graph of a rational function ♦ finding the non-vertical asymptote to the graph of a rational function	For example: ♦ $f(x)=\frac{x^2+2x+4}{x-1}$ ♦ $f(x)=\frac{x}{x^2+1}$, $f(x)=\frac{x^2}{x^2+1}$ Candidates should understand that when the degree of the numerator of the rational function exceeds that of the denominator by 1, non-vertical asymptotes occur.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Investigating features of graphs and sketching graphs of functions	♦ investigating points of inflection ♦ investigating features of graphs: ▪ points of inflection ▪ stationary points ▪ domain and range ▪ odd, even, or neither ▪ continuous or discontinuous ▪ extrema of functions: the maximum and minimum values of a continuous function *f* defined on a closed interval $[a,b]$ can occur at stationary points, end points, or points where f' is not defined ♦ sketching graphs using features given or obtained ♦ sketching related functions: ▪ modulus functions ▪ inverse functions ▪ functions differentiated ▪ translations and reflections	Candidates should be aware that points of inflection occur where: ♦ the function is defined at the point ♦ the second derivative is 0 or undefined ♦ there is a change in concavity For example: ♦ Establish the coordinates of the point of inflection on the graph of $y = x^3 + 3x^2 + 2x$. ♦ Calculate the maximum value, $0 \le x \le 4$, of $f(x) = e^x \sin^2 x$. ♦ Sketch the graph of $y = \frac{x}{x^2 - 1}$. ♦ Given $f(x)$, sketch the graph of $y = \lvert f(x) + a \rvert$ (i) $y = \lvert 2f(x) - 1 \rvert$ (ii) $y = \lvert 5 - f(x) \rvert$

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Expanding expressions using the binomial theorem	♦ using the binomial theorem $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, for $r, n \in \mathbb{N}$ to expand an expression of the form $(ax^p + by^q)^n$, where $a, b \in \mathbb{Q}$; $p, q \in \mathbb{Z}$; $n \le 7$ ♦ using the general term for a binomial expansion, finding a specific term in an expression	For example: ♦ Expand $\left(3x - \frac{1}{2x}\right)^6$. ♦ Find the coefficient of x^7 in $\left(\frac{2}{x} + x\right)^{11}$. ♦ Find the term independent of x in the expansion of $\left(3x^2 - \frac{2}{x}\right)^9$.
Finding the general term and summing arithmetic and geometric progressions	♦ applying the rules of sequences and series to find: ▪ the *n*th term ▪ the sum to *n* terms ▪ common difference of arithmetic sequences ▪ common ratio of geometric sequences ♦ determining the sum to infinity of geometric series ♦ determining the condition for a geometric series to converge	For example, $1 + 2x + 4x^2 + 8x^3 + \cdots$ has a sum to infinity if and only if $\lvert x \rvert < \frac{1}{2}$.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Applying summation formulae	♦ knowing and using sums of certain series, and other straightforward results and combinations of these	For example: ♦ $\sum_{r=1}^{n} r,\ \sum_{r=1}^{n} r^2,\ \sum_{r=1}^{n} r^3$ ♦ $\sum_{r=1}^{n}(ar+b) = a\sum_{r=1}^{n} r + \sum_{r=1}^{n} b = \frac{an(n+1)}{2} + bn$ ♦ $\sum_{r=k+1}^{n} f(r) = \sum_{r=1}^{n} f(r) - \sum_{r=1}^{k} f(r)$
Using the Maclaurin expansion to find specified terms of the power series for simple functions	♦ using the Maclaurin expansion to find a power series for simple functions ♦ combining Maclaurin expansions to find a power series	For example: ♦ the first three terms of $\frac{1}{1+x^2}$ ♦ $e^{\sin x}$, up to, and including, the term in x^3 ♦ the first four non-zero terms of $e^x \sin 3x$ Candidates should be familiar with the standard power series expansions of e^x, $\sin x$, $\cos x$ and $\ln(1 \pm x)$. Candidates could discuss conditions for convergence.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Disproving a conjecture by providing a counterexample	♦ disproving a conjecture by providing a counterexample ♦ knowing and using the symbols $\exists$ (there exists) and $\forall$ (for all) ♦ giving the negation of a statement	For example, for all real values of a and b, $a-b>0 \Rightarrow a^2-b^2>0$. A counterexample is $a=3,\ b=-4$.
Using indirect or direct proof in straightforward examples	♦ proving a statement by contradiction ♦ using proof by contrapositive ♦ using direct proof in straightforward examples	In this area of the course, candidates need to think clearly and communicate their findings clearly. For example, let n be an integer. Prove by contradiction that if n^2 is even, then n is even. Let n^2 be even. Suppose n is odd. $n=2k+1,\ \ k\in\mathbb{Z}$ Then $n^2=4k^2+4k+1$ $n^2=2\left(2k^2+2k\right)+1$ Therefore, n^2 is odd. This is a contradiction as n^2 is even. Therefore, the original statement is true. For example, let n be an integer. Prove by contrapositive that if n^2 is even, then n is even.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
		The contrapositive of the given statement is, 'If n is odd, then n^2 is odd'. Let n be odd. Then, $n = 2k+1,\ \ k \in \mathbb{Z}$ $n^2 = 4k^2 + 4k + 1$ $n^2 = 2\left(2k^2 + 2k\right) + 1$ Therefore, n^2 is odd. The contrapositive statement is true. Therefore, the original statement is true. For example, prove directly that the product of an even function and an odd function is an odd function. Let $f(x)$ be an even function and $g(x)$ be an odd function. We have $f(-x) = f(x)$ and $g(-x) = -g(x)$. Let $h(x) = f(x)g(x)$, so that $h(-x) = f(-x)g(-x)$ $= f(x)\left[-g(x)\right]$ $= -f(x)g(x)$ $= -h(x)$ Therefore, $h(x)$ is an odd function.

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
		Candidates would benefit from exposure to proofs that $\sqrt{2}$ is irrational and the infinitude of primes. Direct proof features prominently throughout the course and could include: ♦ standard results in differentiation from first principles ♦ chain rule, product rule, quotient rule ♦ other standard derivatives ♦ integration by substitution ♦ integration by parts ♦ triangle inequality ♦ the sum of first n natural numbers ♦ the sum to n terms of arithmetic and geometric series ♦ standard results in the algebra of vectors and matrices
Using proof by induction	♦ using proof by induction	In this area of the course candidates need to think clearly and communicate their findings clearly. For example: $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$ show that $1+2+2^2+\cdots+2^n = 2^{n+1}-1,\ \forall n \in \mathbb{N}$ 8^n is a factor of $(4n)!,\ \forall n \in \mathbb{N}$ $8^n + 3^{n-2}$ is divisible by 5 for all integers, $n \geq 2$

Algebra, proof and number theory		
Skill	**Explanation**	**Examples**
Using Euclid's algorithm to find the greatest common divisor of two positive integers	♦ using Euclid's algorithm to find the greatest common divisor of two positive integers, for example using the division algorithm repeatedly ♦ expressing the greatest common divisor (of two positive integers) as a linear combination of the two ♦ expressing integers in bases other than 10 ♦ knowing and using the fundamental theorem of arithmetic	For example: ♦ Express 125_8 in base 3.

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
Using Gaussian elimination to solve a 3 × 3 system of linear equations	♦ finding the solution to a system of equations $A\mathbf{x} = \mathbf{b}$, where A is a 3 × 3 matrix and where the solution is unique — candidates should understand the term 'augmented matrix' ♦ showing that a system of equations has no solutions (inconsistency) ♦ showing that a system of equations has an infinite number of solutions (redundancy) ♦ comparing the solutions of related systems of two equations in two unknowns and recognising ill-conditioning	Candidates should be able to solve a 3 × 3 system of linear equations using Gaussian elimination on an augmented matrix. When solving a system of equations, candidates should use elementary row operations to reduce the matrix to triangular form. This approach can also be used to explore situations where the system of equations is inconsistent or redundant. Learning and teaching contexts could include vectors and the different ways three planes can intersect.
Understanding and using matrix algebra	♦ performing matrix operations (at most order three): addition, subtraction, multiplication by a scalar, multiplication of matrices ♦ knowing and applying the properties of matrix addition and multiplication: ▪ $A+B=B+A$ (addition is commutative) ▪ $AB \neq BA$ (multiplication is not commutative in general) ▪ $(A+B)+C=A+(B+C)$ (associativity) ▪ $(AB)C=A(BC)$ (associativity) ▪ $A(B+C)=AB+AC$ (addition is distributive over multiplication)	Candidates should understand the terminology associated with matrices: ♦ element ♦ row ♦ column ♦ order ♦ identity matrix ♦ inverse ♦ determinant ♦ singular ♦ non-singular

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
	♦ knowing and applying key properties of the transpose, the identity matrix, and inverse: ▪ $\left(a_{ij}\right)'_{m\times n} = \left(a_{ji}\right)_{n\times m}$ (rows and columns interchange) ▪ $\left(A'\right)' = A$ ▪ $\left(A+B\right)' = A' + B'$ ▪ $\left(AB\right)' = B'A'$ ▪ A square matrix A is orthogonal if $A'A = AA' = I$ ▪ The $n \times n$ identity matrix I_n for any square matrix A , $AI_n = I_n A = A$ ▪ $B = A^{-1}$ if $AB = BA = I$ ▪ $(AB)^{-1} = B^{-1}A^{-1}$	♦ transpose ♦ orthogonality ♦ conformability ♦ invertible ♦ entry ♦ upper triangular and lower triangular ♦ zero matrix
Calculating the determinant of a matrix	♦ finding the determinant of a 2 × 2 matrix and a 3 × 3 matrix ♦ determining whether a matrix is singular ♦ knowing and applying $\det(AB) = \det A \det B$	Candidates should understand that a (square) matrix, A, is invertible $\Leftrightarrow \det A \neq 0$.
Finding the inverse of a matrix	♦ knowing and using the inverse of a 2 × 2 matrix ♦ finding the inverse of a 3 × 3 matrix	For example: ♦ exploring links between finding the inverse of a 3 × 3 matrix and solving systems of equations

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
		♦ finding the inverse of a 3 × 3 matrix using elementary row operations, the adjoint or matrix algebra; for example, given B and $AB = kI$, find A^{-1} in terms of B
Using transformation matrices	♦ using 2 × 2 matrices to carry out geometric transformations in the plane — the transformations should include rotations, reflections, and dilatations ♦ applying combinations of transformations	Candidates could explore and derive the various matrices associated with: ♦ anticlockwise or clockwise rotations about the origin ♦ reflection in the axes ♦ reflection in lines $y = \pm x$ ♦ dilations centred at the origin
Calculating a vector product	♦ using a vector product method in three dimensions ♦ evaluating the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	
Working with lines in three dimensions	♦ finding the equation of a line in parametric, symmetric, or vector form, given suitable defining information ♦ finding the angle between two lines in three dimensions ♦ determining whether or not two lines intersect and, where possible, finding the point of intersection	Candidates should be familiar with: **Vector form** The position vector, $\mathbf{r}$, of any point on the line is given by: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ ($\lambda \in \mathbb{R}$), where $\mathbf{a}$ is the position vector of a point on the line and $\mathbf{b}$ is a vector in the direction of the line.

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
		If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the equation of the line can be written in the following forms, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. **Parametric form** $x = a_1 + \lambda b_1$ $y = a_2 + \lambda b_2$, $(\lambda \in \mathbb{R})$ $z = a_3 + \lambda b_3$ **Symmetric form** $\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$ $(= \lambda)$
Working with planes	♦ finding the equation of a plane in vector, parametric, or Cartesian form ♦ finding the point of intersection of a plane with a line that is not parallel to the plane ♦ determining the intersection of two or three planes ♦ finding the angle between a line and a plane, or between two planes	Candidates should be familiar with: **Vector form** The position vector, $\mathbf{r}$, of any point on the plane is given by $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $(\lambda, \mu \in \mathbb{R})$ where $\mathbf{a}$ is the position vector of a point on the plane and $\mathbf{b}$ and $\mathbf{c}$ are non-parallel vectors lying in the plane.

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
		If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$, then the equation of the plane can be written in the following forms, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. **Parametric form** $x = a_1 + \lambda b_1 + \mu c_1$ $y = a_2 + \lambda b_2 + \mu c_2 \quad (\lambda, \mu \in \mathbb{R})$ $z = a_3 + \lambda b_3 + \mu c_3$ **Cartesian form** $n_1 x + n_2 y + n_3 z = d$, which arises from $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$, where $\mathbf{a}$ is the position vector of a point on the plane and $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ is a vector normal to the plane. Candidates should be familiar with using the vector product to obtain a normal to the plane. **Intersection of planes** Two distinct planes: ♦ intersect in a line **or** ♦ are parallel

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
		Three distinct planes together: ♦ intersect in a line ♦ intersect at a point **or** ♦ have no point in common The intersection of three planes, along with work on systems of equations, provides a geometric illustration of redundancy and inconsistency.
Performing algebraic operations on complex numbers	♦ performing the operations of addition, subtraction, multiplication, and division ♦ finding the square root ♦ finding the roots of a cubic or quartic equation with real coefficients when one complex root is given ♦ solving equations involving complex numbers	For example: ♦ $\sqrt{8-6i}$ ♦ solve $z+i=2\bar{z}+1$ ♦ solve $z^2=2\bar{z}$
Performing geometric operations on complex numbers	♦ plotting complex numbers in the complex plane (an Argand diagram) ♦ knowing the definition of modulus and argument of a complex number ♦ converting a given complex number from Cartesian to polar form and vice-versa	**Cartesian form** $z=a+bi$, where $a,b\in\mathbb{R}$

Matrices, vectors, and complex numbers		
Skill	**Explanation**	**Examples**
	♦ using de Moivre's theorem with integer and fractional indices ♦ applying de Moivre's theorem to multiple angle trigonometric formulae ♦ applying de Moivre's theorem to find the *n*th roots of a complex number ♦ interpreting geometrically certain equations or inequalities in the complex plane by sketching or describing a straight line or circle that represents the locus of points that satisfy a given equation or inequality	**Polar form** $z = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$ Candidates should use the principal argument: $-\pi < \theta \leq \pi$. For example: ♦ Expand $\left(\cos\theta + i\sin\theta\right)^4$. ♦ Show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$. ♦ Solve $z^6 = -64$.

Appendix 3: question paper brief

The course assessment consists of two question papers, which assess the:

- ability to use mathematical reasoning skills to think logically, provide justification and solve problems
- ability to use a range of complex concepts
- ability to select and apply complex operational skills
- ability to use reasoning skills to interpret information and to use complex mathematical models
- ability to effectively communicate solutions in a variety of mathematical contexts
- ability to explain and justify concepts through the idea of rigorous proof
- ability to think creatively
- application of skills, without the aid of a calculator, in order to demonstrate candidates' underlying grasp of mathematical concepts and processes

The question papers sample the 'Skills, knowledge and understanding' section of the course specification.

This sample draws on all of the skills, knowledge and understanding from each of the following areas:

- algebraic skills
- calculus skills
- geometric skills
- reasoning skills

Command words are the verbs or verbal phrases used in questions and tasks to ask candidates to demonstrate specific skills, knowledge or understanding. For examples of some of the command words used in this assessment, refer to the past papers and specimen question paper on SQA's website.

The course assessment consists of two question papers:

	Paper 1 (non-calculator)	Paper 2
Time	1 hour	2 hours and 30 minutes
Marks	35	80
Skills	This question paper gives candidates an opportunity to apply numeric, algebraic, geometric, trigonometric, calculus, and reasoning skills, **without the aid of a calculator**. Candidates are required to show an understanding of underlying processes, and the ability to use skills within mathematical contexts in cases where a calculator may compromise the assessment of this understanding.	This question paper gives candidates an opportunity to apply numeric, algebraic, geometric, trigonometric, calculus, and reasoning skills. These skills may be facilitated by using a calculator, as this allows more opportunity for application and reasoning.
Percentage of marks across the papers	Approximately 30–50% of the overall marks relate to calculus. Approximately 20–40% of the overall marks relate to algebra, proof and number theory. Approximately 20–40% of the overall marks relate to matrices, vectors and complex numbers.	
Type of question	Short-answer and extended-response questions	
Type of question paper	Semi-structured question papers: separate question paper and answer booklet. The answer booklet is structured with spaces for answers.	
Proportion of level 'C' questions	Some questions use a stepped approach to ensure that there are opportunities for candidates to demonstrate their abilities beyond level 'C'. Approximately 65% of marks are available for level 'C' responses.	
Balance of skills	Operational and reasoning skills are assessed in both question papers. Some questions assess only operational skills (approximately 65% of the marks), but other questions assess operational and reasoning skills (approximately 35% of the marks).	

Administrative information

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History of changes

Version	Description of change	Date
2.0	Course support notes; skills, knowledge and understanding with suggested learning and teaching contexts; and question paper brief added as appendices.	May 2019

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.